NUMBER SYSTEM, SIMPLIFICATION AND APPROXIMATION

This chapter forms a basis of many other topics in mathematics. Let us begin by understanding various types of numbers.

(1) Natural Numbers: All the counting numbers are called natural number.
Example: 1, 2, 3, 4, 5, ………

- **(a) Even Number:** The numbers which are exactly divisible by 2 are called even numbers.
  Example: 2, 4, 6, 8,…..
- **(b) Odd Number:** The numbers which leave a remainder 1 when divided by 2 are called odd numbers.
  Example: 1, 3, 5, 7,……….
- **(c) Prime Number:** If a number is not divisible by any other number except 1 and itself, it is called a prime number.
  Example: 2, 3, 5, 7, 11,…….
  Co-Primes → Two numbers which have no common factor between them except 1 are said to be co-prime to each other. The two numbers individually may be prime or composite.
  Example: 13 and 29 are co-primes
- **(d) Composite Numbers:** Numbers which are divisible by other numbers along with 1 and itself are called Composite Numbers
  Example: 4, 6, 8, 9, 10,……
  The number 1 is neither prime nor composite.

(2) Whole Numbers: Natural numbers along with '0' form the set of whole numbers.
Example: 0, 1, 2, 3,……..

(3) Integers: All counting numbers and their negatives along with are called integers.
Example: ……..-4,-3,-2,-1,0,1,2,3,4, ……..

(4) Rational and irrational Numbers: Any number which can be expressed in the form of $\frac{p}{q}$,where p and q are integers and q≠0, is a rational number.
Example: $\frac{3}{5}$, 4,-6, etc.
Number which are represented by non-terminating and non-recurring decimals are called irrational numbers.
Example: $\sqrt{2}$ = 1.414…….,$\sqrt{3}$ = 1.732……

(5) Real Numbers: Rational and irrational numbers taken together are called real numbers.

Some important formula:

1. $a^2-b^2 = (a + b)(a - b)$
2. $(a + b)^2 = a^2 + b^2 + 2ab$
3. $(a-b)^2 = a^2 + b^2 - 2ab$
4. $(a + b + c)^2 = a^2+b^2+c^2+2ab+2bc+2ac$
5. $(a + b)^3 = a^3+b^3+3ab(a + b)$
6. $(a-b)^3 = a^3-b^3-3ab(a-b)$

7. $a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$

8. $a^3 - b^3 = (a-b)(a^2 + b^2 + ab)$

9. $a^3+b^3+c^3 - 3abc = (a + b + c)(a^2+b^2+c^2-ab-bc-ac)$

Test of divisibility

Divisibility by 2: A number is divisible by 2 if its unit digit is zero or an even number.

Example: 248, 130

Divisibility by 3: A number is divisible by 3 if the sum of its digit is divisible by 3.

Example: 279 ➔ 2+7+9= 18.
18 is divisible by 3 , hence 279 is divisible by 3.

Divisibility by 4: A number id divisible by 4 if the number formed by its last two digits is divisible by 4.

Example: 236784
Here, 84 is divisible by 4, hence 236784 is divisible by 4.

Divisibility by 5: A number is divisible by 5 if the number or its unit digit is either 5 or 0.

Example: 115, 240 etc.

Divisibility by 6: A number is divisible by 6 if it is divisible by both 2 and 3.

Example: 318, 396, etc.

Divisibility by 8: A number is divisible by 8 if the number formed by its last 3 digit is divisible by 8.

Example: 23816.
Here, 816 is divisible by 8, hence 23816 is divisible by 8.

Divisibility by 9: A number is divisible by 9 if the sum of all its digits is divisible by 9.

Example: 72936 ➔ 7+2+9+3+6=27
27 is divisible by 9, hence 72936 is divisible by 9.

Divisibility by 11: A number is divisible by 11 if the difference of the sum of the alternate digits starting from the units digit and the sum of the alternate digits starting from the tens digit is either '0' or is a multiple of 11

Example: 1331
$(1+3)-(3+1) = 0 \Rightarrow$ 1331is divisible by 11

Divisibility by 19: A number is divisible by 19 if the sum of the number formed by digits other than the unit digit and twice the unit digit is divisible by 19.

Example: 76 $\Rightarrow$ $7 + (2\times6) = 19$
Therefore 76 is divisible by 19

Least Common Multiple (LCM)

LCM of two or more numbers is the least number which is divisible by each of these numbers.

Finding LCM

Write the numbers as product of prime factors. Then multiply the product of all the prime factors of the first number by those prime factors of the second number which are not common to the prime factors of the first number. The product is then multiplied by those prime factors of the third number which are not common to the prime factors of the first two numbers.

The final product after considering all the numbers will be the LCM of these numbers.

Example: Find the LCM of 540 and 108?

$540 = 2\times27\times10 = 2^2\times3^2\times5$

$108 = 2^2 \times 3^3$

$LCM = 2^2\times 3^3\times5 = 4\times27\times5 = 540$

Finding LCM by division

Choose one prime factor common to at least two of the given number write the given numbers in a row and divide them by the above prime number. Write the quotient for each number under the number itself. If a number is not divisible by the prime factor selected, write the number as it is Repeat this process until you get quotients which have no common factor.

The product of all the divisors and the numbers in the last line will be the LCM.

Example: Find the LCM of 36, 84 and 90

3	36,	84,	90
3	12,	28,	30
2	4,	28,	10
2	2,	14,	5
	1,	7,	5

LCM $=3\times3\times2\times2\times7\times5= 1260$

Highest Common Factor (HCF)

HCF is the largest factor of two or more given numbers.

HCF is also called Greatest Common Divisor (GCD).

Finding HCF by Factorization method

Express each given number as a product of primes factors. The product of the prime factors common to all the numbers will be the HCF.

Example: Find the HCF of 144, 336 and 2016?

$144=12\times12=3\times2^2\times3\times2^2=3^2\times2^4$

$336 =2^4 \times3\times7$

$2016 = 2^5\times7\times3^2$

HCF $= 3\times2^4= 48$

Finding HCF by Division method

Divide the greater number by the smaller number .Then divide the divisor by the remainder. Now, divide the second divisor by the second remainder

We repeat this process till no remainder is left. The last divisor is the HCF.

Then using the same method, find the HCF of this HCF and the third number. This will be the HCF of the three numbers

Example: HCF of 144,336

```
144)336(2
    288
    ----
     48)144(3
        144
        ---
          0
```

HCF = 48

LCM and HCF of fractions:

LCM of Fractions = $\frac{\text{LCM of Numerators}}{\text{HCF of Denominators}}$

HCF of fractions = $\frac{\text{HCF of Numerators}}{\text{LCM of Denominators}}$

Simplification

BODMAS Rule

This rule depicts the correct sequence in which the operations are to be executed, so to find out the value of a given expression.

B	→	Bracket
O	→	Of
D	→	Division
M	→	Multiplication
A	→	Addition
S	→	Subtraction

Thus is simplifying an expression, first of all the brackets must be removed, strictly in the order (),{},[].
After removing the brackets, we must use the following operations strictly in the order:
(i) of (ii) Division (iii) Multiplication (iv) Addition (v) Subtraction

Approximation

One needs to solve the questions of approximation by taking the nearest approximate values and mark the answers accordingly.

Example: If the given value is 3.009, then the approximate value is 3.

If the given value is 4.45, then the approximate value is 4.50.

Example 1: 2959.85 ÷ 16.001- 34.99 =?

(a) 160 (b) 150 (c) 140 (d) 180 (e) 170

Sol. (b); 2959.85 ÷ 16.001 – 34.99 ≅ 2960 ÷16 – 35 = 150

Example 2: (1702 ÷ 68) × 136.05 =?

(a) 3500 (b) 3550 (c) 3400 (d) 3400 (e) 3525

Sol. (d); (1702 ÷68) × 136.05 ≅ (1700 ÷ 68) ×136 = 3400

Some shortcuts and tricks for calculations

Multiplication by a number close to 10, 100, 1000, etc

Example: 999=1000-1; 101 = 100+1

To multiply with such numbers, convert the number into the form of (10 ±C) or (100±C) etc.

Example: 46×98= 46 × (100 – 2) = 46×2 =4600 -92=4508

Multiplication by 5 or powers of 5: can be converted into multiplication by 10 or powers of 10 by dividing it by 2 and its powers.

Example: $2345 \times 125 = 2345 \times 5^3 = 2345 \times \left(\frac{10}{2}\right)^3 = \frac{2345000}{8} = 293125$

Square of a number which ends with 5.

1. Last two digits of the square are always 25.
2. To find the number which comes before 25, perform the operation n× (n+1), where n is the digit before 5 in the original number.
3. Put the number received in step 2 before 25 and you get the square.

Example: $(65)^2 = ?$

1. Last two digits are 25.
2. The digit before 5 is 6 perform n (n+1) operation on this = 6× (6+1) =6×7 =42
3. Hence the square of 65 will be 4225.

Square of a number containing repeated 1's

1. Count the number of digits. Let the count be n.
2. Now, starting from 1, write the number till n.
3. Then, starting from n write the number till 1.

Example: Find the square of 1111?

Sol. There are four 1's. Now we write numbers from 1 to 4. Then again from 4 to 1. So, $(1111)^2 = 1234321$

Multiplying 2 –digit numbers where the unit's digits add upto 10 and ten's digits are same

Example: 42×48 =?

1. First multiply the unit digits of the numbers. 2×8 =16
2. Then multiply 4 by (4+1) ⇒4×5 =20.
3. The answer is 2016.

Multiplying numbers just over/below 100

Example: 108 × 109 = 11772

The answer is in two parts: 117 and 72.

117 is (108 + 9 or (109 + 8), and 72 is 8×9

New, check for 107 × 106 = 113 42

113 → (107+6) Or (106+7)

42 → (7×6)

Multiplication of a 2-digit number by a 2-digit number

Example: 12×13?

Sol. Steps:

1. Multiply the right-hand digits of multiplicand and multiplier (unit-digit of multiplicand with unit-digit of the multiplier).

```
1  2
   ↕
1  3
-------
   6(2×3)
```

2. Now, do cross-multiplication, i.e., multiply 3 by 1 and 1 by 2. Add the two products and write down to the left of 6.

```
1     2
   ╳
1     3
---------
     56
```

3. In the last step we multiply the left-hand figures of both multiplicand and multiplier (ten's digit of multiplicand with ten's digit of multiplier).

```
 1  2
 ↕
 1  3
--------
  156 (1×1)
```

So, the answer is 156.

Example: 325 × 17 =?

Sol. Steps:

1.
```
3 2 5
    ↕
  1 7
------
    5
```
(5 ×7 =35, put down 5 and carry over 3)

2.
```
3   2   5
        ↕
    1   7
---------
    2   5
```
(2×7 +5×1+3 = 22, put down 2 and carry over 2)

3.
```
3   2   5
  ↖ ↕ ↘
    1   7
---------
    2  25
```
(3×7+2×1+2=25, put down 5 and carry over 2)

4.
```
3   2   5
 ↘
    1   7
---------
55  2   5
```

So, answer is 5525

Multiplication of a 3-digit number by a 3-digit number

Example: 321×132 =

Sol. Steps:

1.
```
3  2  1
      ↕
1  3  2
--------
      2(1×2=2)
```

2.
```
3   2   1
     ╳
1   3   2
---------
   72 (2×2+3×1=7)
```

3.
```
3   2   1
   ╳ ↕ ╳
1   3   2
---------
```
(2×3+3×2+1×1 = 13, write down 3 and carry over 1)

4.
```
3   2   1
  ╳
1   3   2
---------
2372
```
(3×3+1×2+1 =12, write down 2 and carry over 1)

5.
```
3   2   1
↕
1   3   2
---------
4 2 3 7
```
2(1×3+1=4) So, answer is 42372.

Some more short tricks:

(1) 2 +22 + 222 + 2222 = 2 (1+11+111+1111)

2(1234) +2468

(2) 0.2+.022+.0222+.02222.022222=2(0.1+0.11+0.111+0.1111+0.11111=2(0.54321)=1.08642

(3) 2 + 8 + 22 + 88+ 222 + 888 +2222 + 8888 + 22222 +88888 =2(12345)+ 8 (12345)
= (12345) (2+8) =12345 × 10 =123450

(4) $(2222)^2 = 2^2 \times (1111)^2 = 4 \times (1234321) = 4937284$

(5) If unit digit in each number is 5 and difference of the numbers is 10, then they are multiplied as:

Example: (1) 65×75= 48 | 75 → 48 = (6×8), 75 = (Constant) (2) 125×135= 168 | 75 → 168 = (12×14), 75 = (Constant)

Percentage – fraction conversion:

The following percentage values of corresponding fractions must be on your tips:

Example: $62\frac{1}{2}$% of 256 can be easily calculated if we know the fractional value of $62\frac{1}{2}$ % i.e., $\frac{5}{8}$

$1 = 100\%$, $\frac{1}{2} = 50\%$

$\frac{1}{3} = 33\frac{1}{3}\%$, $\frac{2}{3} = 66\frac{2}{3}\%$

$\frac{1}{4} = 25\%$, $\frac{3}{4} = 75\%$

$\frac{1}{5} = 20\%$, $\frac{2}{5} = 40\%$, $\frac{3}{5} = 60\%$, $\frac{4}{5} = 80\%$

$\frac{1}{6} = 16\frac{2}{3}\%$, $\frac{5}{6} = 83\frac{1}{3}\%$

$\frac{1}{7} = 14\frac{2}{7}\%$, $\frac{2}{7} = 28\frac{4}{7}\%$, $\frac{3}{7} = 42\frac{6}{7}\%$, $\frac{4}{7} = 57\frac{1}{7}\%$, $\frac{5}{7} = 71\frac{3}{7}\%$, $\frac{6}{7} = 85\frac{5}{7}\%$

$\frac{1}{8} = 12\frac{1}{2}\%$, $\frac{3}{8} = 37\frac{1}{2}\%$, $\frac{5}{8} = 62\frac{1}{2}\%$, $\frac{7}{8} = 87\frac{1}{2}\%$

$\frac{1}{9} = 11\frac{1}{9}\%$, $\frac{3}{4} = 75\%$, $\frac{4}{9} = 44\frac{4}{9}\%$, $\frac{5}{9} = 55\frac{5}{9}\%$, $\frac{7}{9} = 77\frac{7}{9}\%$, $\frac{8}{9} = 88\frac{8}{9}\%$

$\frac{1}{11} = 9\frac{1}{11}\%$, $\frac{2}{11} = 8\frac{2}{11}\%$, $\frac{3}{11} = 27\frac{3}{11}\%$, $\frac{4}{11} = 36\frac{4}{11}\%$, $\frac{5}{11} = 45\frac{5}{11}\%$

$\frac{1}{20} = 5\%$, $\frac{1}{9} = 5 + \frac{5\times5}{100} = 5.25\%$, $\frac{1}{21} = 5 - \frac{5\times5}{100} = 4.75\%$

Similarly,

$\frac{1}{25} = 4\%$, $\frac{1}{24} = 4 + \frac{4\times4}{100} = 4.16\%$, $\frac{1}{26} = 4 - \frac{4\times4}{100} = 3.84\%$, $\frac{1}{11} = 9.09\%$, $\frac{1}{7} = 14.2857\%$, $\frac{1}{22} = 4.54\%$, $\frac{1}{14} = 7.14\%$

$\frac{1}{33} = 3.03\%$, $\frac{1}{28} = 3.57\%$

Finding the unit place digit when a number is raised to some power

1. When the unit digit of a number is 0, 1, 5 or 6 then on raising that number to any power, the new number obtained will have its unit digit 0, 1, 5 or 6 respectively.
2. When the unit digit of a number is 2:
   Example: $(122)^{159}$
   Divide 159 by 4
   $\frac{159}{4} \rightarrow$ remainder = 3
   (Unit digit of $122)^3 = 2^3 = 8$
   So, the unit digit $(122)^{159} = 8$
3. When the unit digit of the number is 3.
   Example: $(53)^{145}$
   Sol. $\frac{145}{4} \Rightarrow$ Remainder =1
   $3^1 = 3$
   So, unit digit $(53)^{145}$ is 3.
4. When the unit digit is 4.
   Example: 144
   If it is raised to an odd power →Example: $(144)^{145}$, then unit place is 4.
   If it is raised to an even power→ Example: $(144)^{144}$, then unit place is 6.
5. When the unit digit is 7:
   Example: $(327)^{329}$
   Sol: $329 \div 4 \Rightarrow$ rem. = $1 \Rightarrow \quad 7^1 = 7 \quad \Rightarrow$ so, unit digit =7
6. When the unit is 8:
   Example: $(88)^{178}$
   Sol. $178 \div 4 \Rightarrow$ Rem. $=2 \Rightarrow \; 8^2 = 6\underline{4} \quad \Rightarrow$ so, unit digit of $(88)^{178}$ is 4
7. When the unit digit is 9:
   Example: 119
   If it is raised to an odd number→
   Example: → $(119)^{119} \Rightarrow$ unit digit = 9
   If it is raised to an even power→
   Example: → $(119)^{118} \Rightarrow$ unit digit = 1

Finding minimum and maximum values in fractions:

Example: Find maximum value:

$\frac{5}{7}, \frac{9}{4}, \frac{8}{13}, \frac{14}{15} \Rightarrow$

Let us consider $\frac{5}{7}$ and $\frac{9}{4}$

$\frac{5}{7} \times \frac{9}{4}$

$5 \times 4 < 9 \times 7 \Rightarrow \frac{5}{7} < \frac{9}{4}$

Now, let us take: $\frac{9}{4}$ and $\frac{8}{13}$

$\frac{9}{4} \times \frac{8}{13}$

$13\times 9 > 4\times 8 \Rightarrow \frac{9}{4} > \frac{8}{13}$

$\frac{9}{4}$ is greater than both $\frac{5}{7}$ and $\frac{8}{13}$

$\frac{9}{4}$ ⤫ $\frac{14}{15}$

$15\times 9 > 4\times 14 \Rightarrow \frac{9}{4} > \frac{14}{15}$

So, $\frac{9}{4}$ is the greatest value among all given values.

QUESTIONS

1. 8796×223+8796×77=?
(a) 2736900 (b) 2738800 (c) 2658560
(d) 2716740 (e) None of these

2. 1260÷ 14 ÷9=?
(a) 9 (b) 10 (c) 81
(d) 810 (e) None of these

3. 6×3 (3-1) is equal to
(a) 19 (b) 20 (c) 36
(d) 53 (e) None of these

4. 320% of 40 =?
(a) 128 (b) 140 (c) 180
(d) 60 (e) 120

5. 69.69 - 51.54 + 73.64 = ? + 32.42
(a) 47.44 (b) 53.88 (c) 58.38
(d) 44.34 (e) None of these

6. 14.28% of 49 = ?
(a) 8 (b) 11 (c) 7
(d) 16 (e) 15

7. $1\frac{1}{3} - 1\frac{1}{9} + 1\frac{1}{6} = ?$
(a) $1\frac{5}{18}$ (b) $1\frac{7}{18}$ (c) $1\frac{1}{9}$
(d) $1\frac{4}{9}$ (e) None of these

8. 3/7 of 49/6 of 4/7 = ?
(a) 1 (b) 2 (c) 3
(d) 4 (e) None of these

9. 25% of 48 + 50% of 120 = ?% of 1200
(a) 4 (b) 5 (c) 6
(d) 8 (e) 16

10. $\sqrt{52 \times 27 \times 6 + 26 - 4}$ =?
(a) $\sqrt{24}$ (b) $(16)^2$ (c) 24
(d) $\sqrt{16}$ (e) None of these

11. 65% of 240 + /% of 150 = 210
(a) 45 (b) 46 (c) 32
(d) 36 (e) None of these

12. $4\frac{4}{5} \div 6\frac{2}{5} = ?$
(a) 3/4 (b) 5/7 (c) 7/11
(d) 5/8 (e) None of these

13. 26.5% of 488 = ?
(a) 205.65 (b) 211.72 (c) 145.67

(d) 129.32 (e) None of these

14. 140% of 56 + 56% of 140 =?

(a) 78.4 (b) 158.6 (c) 156.8
(d) 87.4 (e) None of these

15. $\frac{16}{24} + \frac{4}{10} - \frac{1}{6}$ =?

(a) $\frac{9}{10}$ (b) $\frac{7}{10}$ (c) $\frac{5}{10}$
(d) $\frac{3}{10}$ (e) None of these

16. 8000 ÷ 16 – 200 =? x 6

(a) 75 (b) 60 (c) 50
(d) 25 (e) None of these

17. 73 × 18 + 486 =? +$(13)^2$

(a) 1485 (b) 1631 (c) 1525
(d) 1225 (e) None of these

18. $\frac{1}{8}$ th of $\frac{6}{7}$ th of 11200 = ?

(a) 1100 (b) 1220 (c) 1430
(d) 1200 (e) None of these

19. (6900÷15) × (468 ÷ 18) =?

(a) 12161 (b) 12116 (c) 14000
(d) 13342 (e) None of these

20. $\frac{3}{5}$ th of 24% of 500 – 32 =?

(a) 20 (b) 30 (c) 50
(d) 40 (e) None of these

Solutions:

1. (e): (8796×223+8796×77) = 8796×(223+77) [By distributive law]
= (8796×300) = 2638800
2. (b): 1260÷14÷9=$\left(1260 \times \frac{1}{14} \times \frac{1}{9}\right)$ =10
3. (c): 6×3 (3-1) = 6×3(2) = 6×6=36
4. (a): $\frac{320\times40}{100} = 128$
5. (e): 69.69-51.54+73.64=? +32.42
⇒ ? = 59.37
6. (c): 14.28% of 49 = $\frac{1}{7} \times 49$ =7
7. (b): $1\frac{1}{3} - 1\frac{1}{9} + 1\frac{1}{6}$ =(1-1+1) + $\left(\frac{1}{3} - \frac{1}{9} + \frac{1}{6}\right)$
= 1+$\left(\frac{6-2+3}{18}\right)$= 1+$\frac{7}{18} = 1\frac{7}{18}$
8. (b): $\frac{3}{7}$ of $\frac{49}{6}$ of $\frac{4}{7} = \frac{3}{7} \times \frac{49}{6} \times \frac{4}{7} = 2$
9. (c): $\frac{1}{4} \times 48 + \frac{1}{2} \times 120 =$ x% of 1200
12+60=$\frac{x\times1200}{100}$ ⇒ x =$\frac{72\times100}{1200}$ =6
10. (e): $\sqrt{52 \times \frac{27}{6} + 26 - 4}$
$= \sqrt{26 \times 9 + 26 - 4} = \sqrt{256}$ = 16
11. (d): 65% of 240 + x% of 150 = 210
210-65% of 240 = x% of 150
$210 - \frac{65\times240}{100} = \frac{x\times150}{100}$
$\frac{x\times150}{100}$ = 210-156=54 ⇒ x = $\frac{5400}{150}$ = 36
12. (a): $\frac{24}{5} \times \frac{5}{32} = \frac{3}{4}$
13. (d): ? = $\frac{26.5\times488}{100} = \frac{265\times488}{100\times10}$ = 129.32
14. (c): ? = $\frac{140\times56}{100} + \frac{56\times140}{100} = \frac{2\times56\times140}{100}$ = 156.8
15. (a): $\frac{16}{24} + \frac{4}{10} - \frac{1}{6} = \frac{5\times16+12\times4-1\times20}{120}$
$= \frac{80+48-20}{120} = \frac{108}{120} = \frac{9}{10}$
16. (c): 8000÷16-200=?×6
? $= \frac{\frac{8000}{16}-200}{6} = \frac{500-200}{6}$ = 50
17. (b); 73 × 18 + 486=? +$(13)^2$
? = 73 × 18 +486 - 169=1314+317=1631
18. (d): $\frac{1}{8}$ th of $\frac{6}{7}$ th of 11200 =?
? = $\frac{1}{8} \times \frac{6}{7} \times 11200 = \frac{1}{8}$ ×6×1600
= 6×200=1200
19. (b): (6990÷15) ÷ (468 ÷ 18)
= $\frac{6990}{15} \times \frac{468}{18}$ = 466×26=12116
20. (d): $\frac{3}{5} \times \frac{24\times500}{100} - 32$ = ? ⇒ 72-32=40